

Cosmological perturbations and observational constraints on non-local massive gravity

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Non-local massive gravity can provide an interesting explanation for the late-time cosmic acceleration, with a dark energy equation of state w_{DE} smaller than -1 in the past. We derive the equations of linear cosmological perturbations to confront such models with the observations of large-scale structures. The effective gravitational coupling to non-relativistic matter associated with galaxy clusterings is close to the Newton's gravitational constant G for a mass scale m slightly smaller than the today's Hubble parameter H_0 . Taking into account the background expansion history as well as the evolution of matter perturbations δ_m , we test for these models with the Type Ia Supernovae (SnIa) from the Union 2.1, the Cosmic Microwave Background (CMB) measurements from Planck, a collection of baryon acoustic oscillations (BAO), and the growth rate data of δ_m . Using a higher value of H_0 derived from its direct measurement ($H_0 \gtrsim 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$) the data strongly support the non-local massive gravity model ($-1.1 \lesssim w_{\text{DE}} \lesssim -1.04$ in the past) over the Λ CDM model ($w_{\text{DE}} = -1$), whereas for a lower prior ($67 \text{ km s}^{-1} \text{ Mpc}^{-1} \lesssim H_0 \lesssim 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$) the two models are statistically comparable.

I. INTRODUCTION

Modified gravitational theories have received much attention in connection to the dark energy problem [1]. In particular, the recent observational constraints derived from Planck and other data show that the dark energy equation of state w_{DE} smaller than -1 is favored [2]. This may imply the infrared modification of gravity from General Relativity (GR), because the models in the framework of GR—such as quintessence [3] and k-essence [4]—generally predict w_{DE} larger than -1 .

So far many dark energy models based on the large-distance modification of gravity have been proposed—including the Dvali-Gabadadze-Porrati (DGP) model [5], $f(R)$ gravity [6], Brans-Dicke theories [7], and Galileons [8]. In the DGP model the cosmic acceleration can be realized by the gravitational leakage to the fifth dimension, but it suffers from the incompatibility with observations [9] as well as the ghost problem [10]. In $f(R)$ gravity and Brans-Dicke theories it is possible to construct viable dark energy models at the expense of designing scalar potentials to be compatible with both cosmological and local gravity constraints [11]. In covariant Galileons there exist a tracker solution along which w_{DE} evolves from -2 (matter era) to -1 (de Sitter era) [12], but only the late-time tracking solutions are allowed from the joint data analysis of SnIa, CMB, and BAO [13].

Besides the theories mentioned above, massive gravity has recently received significant attention due to a possibility of the late-time cosmic acceleration with a mass scale m of the order of the today's Hubble parameter H_0 . In the original Fierz-Pauli theory [14] there exists a so-called van Dam-Veltman-Zakharov (vDVZ) discontinuity [15] with which the linearized GR cannot be recovered in the $m \rightarrow 0$ limit. In the presence of non-linear interactions the problem of the vDVZ discontinuity can be cured [16], but an instability mode called the Boulware-Deser (BD) ghost appears due to non-linearities [17].

De Rham, Gabadadze and Tolley (dRGT) constructed a massive gravity theory [18] in which the BD ghost is absent. In addition to acausality of the theory [19] and the requirement of an external reference metric, there are some problems when the dRGT theory is applied to the cosmology. On the homogenous and isotropic cosmological background, it was shown that at least one ghost exists among five propagating degrees of freedom [20]. The self-accelerating solutions in the dRGT theory are also unstable against scalar and vector perturbations [21]. The possible way out of these problems is to break the homogeneity or the isotropy of the Universe [22, 23] or to introduce other degrees of freedom [24–26].

An alternative approach to massive gravity was recently suggested by Jaccard *et al.* [27], who introduced non-local terms to obtain fully covariant equations of motion without referring to any external reference metric (see also Refs. [28–36] for related works). This theory—dubbed non-local massive gravity (NLMG)—respects causality and

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reduces to a massless one without the vDVZ discontinuity in the $m \rightarrow 0$ limit. The covariant equations of motion are given by

$$G_{\mu\nu} - m^2(\square^{-1}G_{\mu\nu})^T = 8\pi G T_{\mu\nu}, \quad (1.1)$$

where $G_{\mu\nu}$ is the Einstein tensor, $T_{\mu\nu}$ is the energy-momentum tensor, G is the gravitational constant, and the subscript T represents the extraction of the transverse part.

The background cosmological dynamics based on Eq. (1.1) was studied in Ref. [37]. There is a rapidly growing scalar mode responsible for the late-time cosmic acceleration, in which case the dark energy equation of state evolves from $w_{\text{DE}} = -1.725$ (matter era) to $w_{\text{DE}} = -1.506$ (accelerated era). Since the Planck data combined with the SnIa (SNLS) and WMAP polarization data placed the bound $w_{\text{DE}} = -1.13^{+0.13}_{-0.14}$ (95 % CL) for constant w_{DE} [2], the NLMG model (1.1) is in tension with the current observations of CMB and SnIa. In order to avoid the rapid growth of the scalar mode, we also require that the mass m is much smaller than H_0 .

Alternatively, Maggiore [38] proposed a model given by the field equation

$$G_{\mu\nu} - \frac{1}{3}m^2(g_{\mu\nu}\square^{-1}R)^T = 8\pi G T_{\mu\nu}, \quad (1.2)$$

where $g_{\mu\nu}$ is the metric tensor and R is the Ricci scalar. In this case the strong instability of a scalar mode present in the theory (1.1) is avoided, so that the dark energy equation of state does not significantly deviate from -1 ($w_{\text{DE}} \approx -1.1$ in the deep matter era). The model has a predictive power due to the presence of a single parameter m alone. For the today's dark energy density parameter $\Omega_{\text{DE}}^{(0)} \simeq 0.68$, the mass m is fixed to be $m \simeq 0.67H_0$ [38, 39]. In this case it was also shown that the General Relativistic behavior can be recovered inside the solar system [40].

If we use a quadratic Lagrangian of gravitational waves associated with the perturbation equation of the theory (1.2), the resulting propagator apparently involves a ghost-like massive scalar [38]. Foffa *et al.* [41] showed that this apparent ghost is not a propagating degree of freedom and in the $m \rightarrow 0$ limit it smoothly approaches a non-radiative degree of freedom of GR. This implies that we may regard Eq. (1.2) as an effective classical equation of motion rather than promoting it directly to a full quantum field theory (which typically involves some classical or quantum averaging). The issue of quantization—including ghosts—would be addressed in an underlying fundamental theory with a possible ultraviolet completion.

In this paper we study the cosmology (at the classical level) and observational constraints on the NLMG models. In Sec. II the background equations of motion are derived for general models including (1.1) and (1.2). We then discuss the evolution of w_{DE} as well as the mass scale m constrained from the background cosmology. In Sec. III we obtain the full equations of linear cosmological perturbations for the NLMG model (1.2). We also discuss the behavior of perturbations for the sub-horizon modes relevant to large-scale structures. In Sec. IV we confront the NLMG model (1.2) with the latest observations of SnIa, CMB, BAO, and redshift-space distortions. Sec. V is devoted to conclusions.

II. BACKGROUND EQUATIONS OF MOTION

We start with the following equations of motion of the NLMG models¹

$$G_{\mu\nu} - m^2\square^{-1}(a_1 R_{\mu\nu} + a_2 g_{\mu\nu}R)^T = 8\pi G T_{\mu\nu}, \quad (2.1)$$

where a_1 and a_2 are constants, $R_{\mu\nu}$ is the Ricci tensor, \square is the d'Alembertian operator and \square^{-1} is its inverse computed by using the retarded Green's function due to causality [39]. The model (1.1) corresponds to $a_1 = 1$ and $a_2 = -1/2$, whereas the model (1.2) is characterized by $a_1 = 0$ and $a_2 = 1/3$. We introduce a tensor $S_{\mu\nu}$ obeying the relation

$$\square S_{\mu\nu} = a_1 R_{\mu\nu} + a_2 g_{\mu\nu}R. \quad (2.2)$$

In order to respect the continuity equation $\nabla^\mu T_{\mu\nu} = 0$ of matter in Eq. (2.1), we pick up the transverse part $S_{\mu\nu}^T$ of the symmetric tensor $S_{\mu\nu}$ satisfying $\nabla^\mu S_{\mu\nu}^T = 0$, that is

$$G_{\mu\nu} - m^2 S_{\mu\nu}^T = 8\pi G T_{\mu\nu}. \quad (2.3)$$

¹ Following Ref. [27] we use the metric signature $(-, +, +, +)$ in this paper. Note that the metric signature used in Ref. [37] is $(+, -, -, -)$.

The tensor $S_{\mu\nu}$ can be decomposed as [42]

$$S_{\mu\nu} = S_{\mu\nu}^T + (\nabla_\mu S_\nu + \nabla_\nu S_\mu)/2. \quad (2.4)$$

Let us consider the flat Friedmann-Lemaître-Robertson-Walker (FLRW) spacetime described by the metric $ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j$, where t is the cosmic time. On this background the vector S_μ has a time component S_0 alone, so that

$$(S_0^0)^T = u + \dot{S}_0, \quad (S_i^i)^T = v + 3HS_0, \quad (2.5)$$

where $u \equiv S_0^0$ and $v \equiv S_i^i$, $H \equiv \dot{a}/a$, and a dot represents a derivative with respect to t . For the energy-momentum tensor $T_{\mu\nu}$, we take into account a perfect fluid obeying the continuity equation

$$\dot{\rho} + 3H(\rho + P) = 0, \quad (2.6)$$

where ρ is the energy density and P is the pressure of the fluid. From Eq. (2.1) we obtain the following equations of motion

$$3H^2 + m^2(u + \dot{S}_0) = 8\pi G\rho, \quad (2.7)$$

$$2\dot{H} + 3H^2 + \frac{m^2}{3}(v + 3HS_0) = -8\pi GP. \quad (2.8)$$

From the (00) and (ii) components of Eq. (2.2) it follows that

$$\ddot{u} + 3H\dot{u} - 6H^2u + 2H^2v = -3(a_1 + 4a_2)H^2 - 3(a_1 + 2a_2)\dot{H}, \quad (2.9)$$

$$\ddot{v} + 3H\dot{v} - 2H^2v + 6H^2u = -9(a_1 + 4a_2)H^2 - 3(a_1 + 6a_2)\dot{H}. \quad (2.10)$$

The divergence of Eq. (2.4) gives $2\nabla^\mu S_{\mu\nu} = \nabla^\mu(\nabla_\mu S_\nu + \nabla_\nu S_\mu)$. The $\nu = 0$ component of this equation reads

$$\ddot{S}_0 + 3H\dot{S}_0 - 3H^2S_0 = -(\dot{u} + 3Hu - Hv). \quad (2.11)$$

In order to simplify the analysis, we define

$$U \equiv u + v, \quad V \equiv u - v/3, \quad (2.12)$$

by which $u = (U + 3V)/4$ and $v = (3/4)(U - V)$. The field U corresponds to the trace part of the tensor S_ν^μ , whereas the field V characterizes the difference between the time and spatial diagonal components of S_ν^μ . On the r.h.s. of Eqs. (2.7) and (2.8) we take into account the contribution of non-relativistic matter (density ρ_m , pressure $P_m = 0$) and radiation (density ρ_r , pressure $P_r = \rho_r/3$). We can write Eqs. (2.7) and (2.8) in the following forms

$$3H^2 = 8\pi G(\rho_m + \rho_r + \rho_{DE}), \quad (2.13)$$

$$2\dot{H} + 3H^2 = -8\pi G(P_r + P_{DE}), \quad (2.14)$$

where

$$\rho_{DE} = \frac{m^2}{32\pi G}(4\zeta X - 4X' - U - 3V), \quad P_{DE} = \frac{m^2}{32\pi G}(U - V + 4X), \quad (2.15)$$

and

$$X = HS_0, \quad \zeta = \frac{H'}{H}. \quad (2.16)$$

A prime represents a derivative with respect to $N = \ln(a/a_i)$, where a_i is the initial scale factor. The dark energy equation of state is then given by

$$w_{DE} = \frac{P_{DE}}{\rho_{DE}} = \frac{U - V + 4X}{4\zeta X - 4X' - U - 3V}, \quad (2.17)$$

whereas the effective equation of state is $w_{\text{eff}} = -1 - 2\zeta/3$. From Eqs. (2.9)-(2.11) it follows that

$$U'' + (3 + \zeta)U' = -6(a_1 + 4a_2)(2 + \zeta), \quad (2.18)$$

$$V'' + (3 + \zeta)V' - 8V = -2a_1\zeta, \quad (2.19)$$

$$X'' + (3 - \zeta)X' - (3 + 3\zeta + \zeta')X = -\frac{1}{4}(U' + 3V' + 12V). \quad (2.20)$$

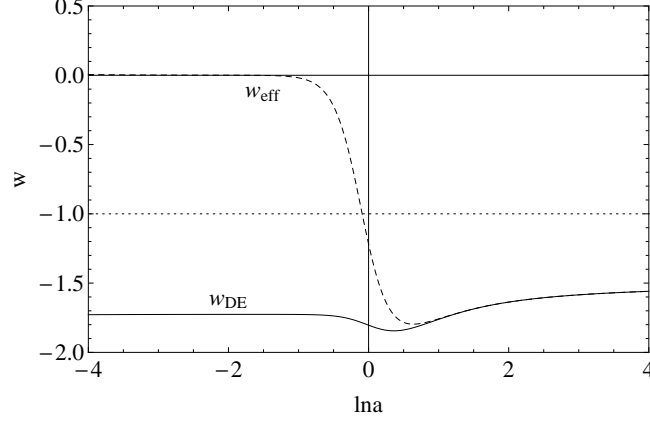


FIG. 1: Evolution of the dark energy equation of state w_{DE} and the effective equation of state w_{eff} versus $\ln a$ for the theory with $a_1 = 0.01$ and $a_2 = 1/3$. The present epoch corresponds to $\ln a = 0$ (i.e., $a = 1$).

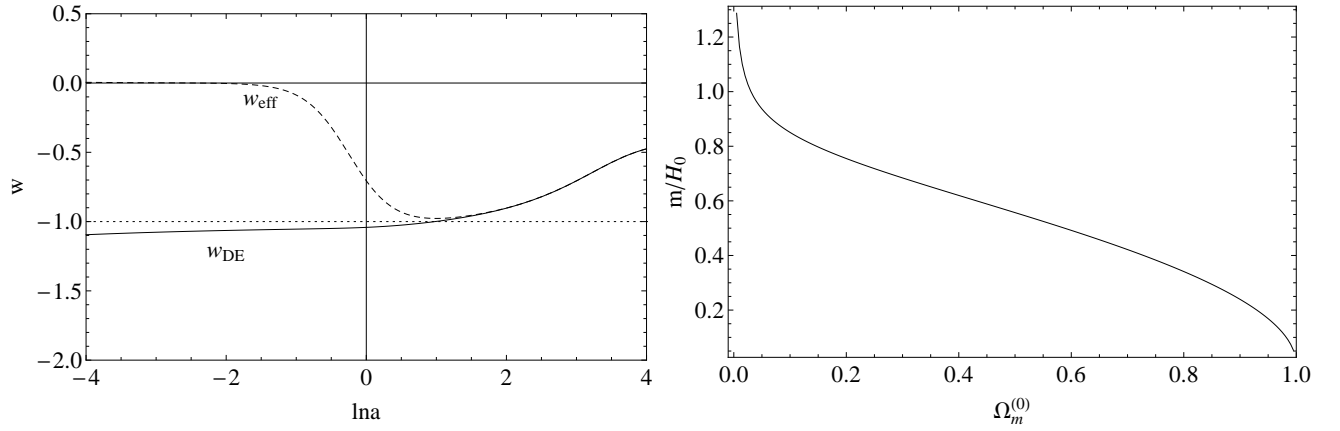


FIG. 2: Left: Evolution of the dark energy equation of state w_{DE} and the effective equation of state w_{eff} versus $\ln a$ for the theory with $a_1 = 0$ and $a_2 = 1/3$. Right: The mass scale m (divided by H_0) versus the today's matter density parameter $\Omega_m^{(0)}$.

From Eq. (2.14) the functions ζ and ζ' obey

$$\zeta = -\frac{3}{2} - \frac{m^2}{8H^2}(U - V + 4X) - \frac{1}{2}\Omega_r, \quad (2.21)$$

$$\zeta' = 2\Omega_r - 3\zeta - 2\zeta^2 - \frac{m^2}{8H^2}(U' - V' + 4X'), \quad (2.22)$$

where we used the fact that the radiation density parameter $\Omega_r = 8\pi G\rho_r/(3H^2)$ satisfies

$$\Omega_r' = -(4 + 2\zeta)\Omega_r. \quad (2.23)$$

From Eq. (2.13) the matter density parameter $\Omega_m = 8\pi G\rho_m/(3H^2)$ is known to be

$$\Omega_m = 1 - \Omega_r - \Omega_{\text{DE}}, \quad \text{where} \quad \Omega_{\text{DE}} = \frac{m^2}{12H^2}(4\zeta X - 4X' - U - 3V). \quad (2.24)$$

The density parameter of radiation today (corresponding to $a = 1$) is given by

$$\Omega_r^{(0)} = \Omega_\gamma^{(0)}(1 + 0.2271N_{\text{eff}}), \quad (2.25)$$

where $\Omega_\gamma^{(0)}$ is the photon density parameter and N_{eff} is the relativistic degrees of freedom. We adopt the standard values $\Omega_\gamma^{(0)} = 2.469 \times 10^{-5} h^{-2}$ and $N_{\text{eff}} = 3.04$, where $H_0 = 100 h \text{ km s}^{-1} \text{ Mpc}^{-1}$ [43]. So, given that $\Omega_r^{(0)}$ is fixed

from the CMB, the only free parameter is $\Omega_m^{(0)}$. From Eq. (2.24) the mass ratio m/H_0 is calculated as

$$\frac{m}{H_0} = \sqrt{\frac{12(1 - \Omega_m - \Omega_r)}{4\zeta X - 4X' - U - 3V}} \Big|_{a=1}. \quad (2.26)$$

We integrate the dynamical equations of motion (2.18)-(2.23) with the initial conditions

$$U(t_i) = \dot{U}(t_i) = V(t_i) = \dot{V}(t_i) = X(t_i) = \dot{X}(t_i) = 0, \quad (2.27)$$

where t_i corresponds to the time at the deep radiation era. The initial conditions can be fixed as above once the definition of the retarded inverse d'Alembertian \square^{-1} is given. For example, the field U is proportional to $(\square^{-1}R)^T = -\int_{t_i}^t dt' a^{-3}(t') \int_{t_i}^{t'} dt'' a^3(t'') R(t'')$ and hence $U(t_i) = \dot{U}(t_i) = 0$ [39]. For given m , we numerically solve the background equations of motion (2.18)-(2.23) and evaluate (2.26) to check the consistency of the solutions.

From Eq. (2.19) it is clear that the field V is unstable for $a_1 \neq 0$ due to the presence of the $-8V$ term. In fact, for the theory with $a_1 = 1$ and $a_2 = -1/2$ [27], this instability leads to the rapid growth of V and X , by which the dark energy equation of state evolves as $w_{\text{DE}} \simeq -1.791$ (radiation era), $w_{\text{DE}} \simeq -1.725$ (matter era), and $w_{\text{DE}} \simeq -1.506$ (accelerated era) [37]. Unless a_1 is very close to 0, the same property also holds for the theories with $a_1 \neq 0$.

In Fig. 1 we plot the evolution of w_{DE} and w_{eff} for $a_1 = 0.01$ and $a_2 = 1/3$ with the today's matter density parameter $\Omega_m^{(0)} = 0.323$. As in the theory with $a_1 = 1$ and $a_2 = -1/2$, the dark energy equation of state during the matter-dominated epoch is around $w_{\text{DE}} \simeq -1.725$ and it finally approaches the value -1.506 . In this case the mass m is much smaller than H_0 , which is required to avoid the early dominance of dark energy [37]. The evolution of w_{DE} shown in Fig. 1 is incompatible with the joint data analysis of CMB, SnIa, and BAO, due to the large deviation from -1 . This conclusion holds not only from the WMAP data combined with the SnIa and BAO measurements [13, 44] but also from the Planck data combined with the SnIa and WMAP polarization measurements [2].

When $a_1 = 0$ the r.h.s. of Eq. (2.19) vanishes, so that the field V does not contribute to the cosmological dynamics for the initial conditions given in (2.27). In the deep radiation era ($\zeta \simeq -2$ and $\zeta' \simeq 0$) the field U is almost frozen, but it starts to evolve once ζ deviates from -2 . During the matter-dominated epoch ($\zeta \simeq -3/2$ and $\zeta' \simeq 0$), integrations of Eqs. (2.18) and (2.20) read

$$U \simeq -8a_2 N + c_1, \quad (2.28)$$

$$X \simeq \frac{4}{3}a_2 + c_2 e^{-(9-\sqrt{57})N/4}, \quad (2.29)$$

where c_1 and c_2 are integration constants, and we neglected the decaying-mode solutions. If the field value $|U|$ at the radiation-matter equality (identified as $N = 0$) is much smaller than 1, it follows that $|c_1| \ll 1$. On using the solution (2.29), the dark energy density (2.15) in the matter era is given by

$$\rho_{\text{DE}} \simeq \frac{m^2}{4\pi G} a_2 \left(N - 1 - \frac{c_1}{8a_2} \right), \quad (2.30)$$

where the term $c_1/(8a_2)$ is much smaller than 1 for $|a_2| = O(1)$. In the regime $N \gtrsim 1$, ρ_{DE} is positive for

$$a_2 > 0. \quad (2.31)$$

This condition is assumed in the following discussion.

Neglecting the second term on the r.h.s. of Eq. (2.29), the dark energy equation of state (2.17) in the deep matter era reads

$$w_{\text{DE}} \simeq -\frac{1-16r}{1-24r}, \quad r \equiv -\frac{a_2}{3U}. \quad (2.32)$$

In the regime $N \gtrsim 1$ we have $U \simeq -8a_2 N$ and $r \simeq 1/(24N)$ for $a_2 = O(1)$, so that $w_{\text{DE}} \simeq -[1 - 2/(3N)]/(1 - 1/N)$. Hence the dark energy equation of state (2.32) is in the range $-1.11 < w_{\text{DE}} < -1$ for $N > 4$. At late times ($N \gg 1$) the terms m^2/H^2 grow in Eqs. (2.21) and (2.22), so Eq. (2.32) starts to lose its validity.

In the left panel of Fig. 2 the evolution of w_{DE} and w_{eff} is plotted for $a_1 = 0$, $a_2 = 1/3$, and $\Omega_m^{(0)} = 0.323$. The value $a_2 = 1/3$ was chosen to match with the one in Ref. [38], but its precise value does not matter provided $a_2 = O(1)$. As estimated above, the dark energy equation of state exhibits mild growth around $-1.1 < w_{\text{DE}} < -1.04$ by today ($-4 < \ln a < 0$). This mild variation of w_{DE} is followed by more rapid growth toward $w_{\text{DE}} \approx -0.5$ in the future. In this case the mass is found to be $m/H_0 \simeq 0.67$, which shows good agreement with the one derived in Ref. [38]. In the right panel of Fig. 2 we also plot the mass ratio m/H_0 versus $\Omega_m^{(0)}$ in the range $\Omega_m^{(0)} \in [0, 1]$. For larger $\Omega_m^{(0)}$ the mass m gets smaller, but it is typically of the order of $m/H_0 = O(0.1)$.

III. COSMOLOGICAL PERTURBATIONS

In this section we shall derive the equations of motion for linear cosmological perturbations for the NLMG model (2.1) with $a_1 = 0$. From Eq. (2.2) we have $\square S_\nu^\mu = a_2 \delta_\nu^\mu R$ and hence

$$\square \hat{U} = 4a_2 R, \quad (3.1)$$

$$S_{\mu\nu} = \frac{1}{4} g_{\mu\nu} \hat{U}, \quad (3.2)$$

where $\hat{U} \equiv S_0^0 + S_i^i$. On the flat FLRW background the field \hat{U} is identical to U introduced in Eq. (2.12), with $V = 0$. From Eqs. (2.4) and (3.2) the field equations of motion (2.3) read

$$G_{\mu\nu} - m^2 \left[\frac{1}{4} g_{\mu\nu} \hat{U} - \frac{1}{2} (\nabla_\mu S_\nu + \nabla_\nu S_\mu) \right] = 8\pi G T_{\mu\nu}. \quad (3.3)$$

Taking the covariant derivative of Eq. (2.4), we obtain

$$\nabla_\nu \hat{U} = 2 (\square S_\nu + \nabla_\mu \nabla_\nu S^\mu). \quad (3.4)$$

We decompose the field \hat{U} into the background component $U(t)$ and the perturbation $\delta U(t, \mathbf{x})$, as

$$\hat{U} = U(t) + \delta U(t, \mathbf{x}). \quad (3.5)$$

The time component of the vector S_μ can be also decomposed as $S_0 = \bar{S}_0(t) + \delta S_0(t, \mathbf{x})$, where we omit the bar in the following for simplicity. The spatial component of S_μ can be written as $S_i = S_i^T + \partial_i \delta S$, where δS is a scalar and S_i^T is a transverse vector satisfying $\partial_i S_i^T = 0$. Since we are interested in only scalar perturbations, we do not consider the contribution of vector perturbations S_i^T . Then, the four-vector S_μ can be expressed as

$$S_\mu = (S_0 + \delta S_0, \partial_i \delta S). \quad (3.6)$$

In order to derive the full perturbation equations of motion, we need to expand Eqs. (3.1), (3.3), and (3.4) up to first order in perturbations. In doing so, we consider scalar metric perturbations Φ and Ψ described by the following metric in longitudinal gauge [45]:

$$ds^2 = -(1 + 2\Phi)dt^2 + a^2(t)(1 - 2\Psi)\delta_{ij}dx^i dx^j, \quad (3.7)$$

for which the perturbations of the Ricci scalar R and the Einstein tensor $G_{\mu\nu}$ etc can be computed. Since our interest is the evolution of perturbations during the matter era, we take into account a non-relativistic perfect fluid characterized by the energy-momentum tensor:

$$T_0^0 = -(\rho_m + \delta\rho_m), \quad T_i^0 = -\rho_m v_{m,i}, \quad T_j^i = 0 \quad (i, j = 1, 2, 3), \quad (3.8)$$

where $\delta\rho_m$ is the density perturbation and v_m is the velocity potential.

A. Perturbation equations

The perturbation $\delta T^{\mu\nu}$ of the matter energy-momentum tensor $T^{\mu\nu}$ obeys the continuity equation

$$\delta T^{\mu\nu}{}_{;\mu} = 0. \quad (3.9)$$

From the $\nu = 0$ and $\nu = i$ components of Eq. (3.9), we obtain the following equations in Fourier space respectively

$$\delta \dot{\rho}_m + 3H\delta\rho_m - 3\rho_m \dot{\Psi} + \frac{k^2}{a^2} \rho_m v_m = 0, \quad (3.10)$$

$$\dot{v}_m = \Phi, \quad (3.11)$$

where k is a comoving wave number. We introduce the gauge-invariant density contrast

$$\delta_m \equiv \frac{\delta\rho_m}{\rho_m} + 3H v_m. \quad (3.12)$$

Taking the time derivative of Eq. (3.10) and using Eq. (3.11), the density contrast satisfies

$$\ddot{\delta}_m + 2H\dot{\delta}_m + \frac{k^2}{a^2}\Phi = 3\ddot{B} + 6H\dot{B}, \quad (3.13)$$

where $B \equiv \Psi + Hv_m$.

From the (00), (0i), (ij) [$i \neq j$], and the trace of the (ii) parts of the perturbed version of Eq. (3.3), we obtain the following equations of motion in Fourier space respectively:

$$\frac{2k^2}{a^2}\Psi + 6H(\dot{\Psi} + H\Phi) - m^2\left(\frac{1}{4}\delta U + \delta\dot{S}_0 - 2\dot{S}_0\Phi - S_0\dot{\Phi}\right) = -8\pi G\delta\rho_m, \quad (3.14)$$

$$2(\dot{\Psi} + H\Phi) + \frac{m^2}{2}(\delta\dot{S} + \delta S_0 - 2S_0\Phi - 2H\delta S) = 8\pi G\rho_m v_m, \quad (3.15)$$

$$\Psi - \Phi + m^2\delta S = 0, \quad (3.16)$$

$$6\ddot{\Psi} + 6H(\dot{\Phi} + 3\dot{\Psi}) + 6(3H^2 + 2\dot{H})\Phi - 2\frac{k^2}{a^2}(\Phi - \Psi) - m^2\left[\frac{3}{4}\delta U + \frac{k^2}{a^2}\delta S + 3H\delta S_0 - 3S_0(\dot{\Psi} + 2H\Phi)\right] = 0. \quad (3.17)$$

From Eq. (3.1) it follows that

$$\delta\ddot{U} + 3H\delta\dot{U} + \frac{k^2}{a^2}\delta U - 2\Phi(\ddot{U} + 3H\dot{U}) - (\dot{\Phi} + 3\dot{\Psi})\dot{U} = 8a_2\left[3(\ddot{\Psi} + 4H\dot{\Psi} + H\dot{\Phi}) + 6(2H^2 + \dot{H})\Phi + \frac{k^2}{a^2}(2\Psi - \Phi)\right]. \quad (3.18)$$

The $\nu = 0$ and $\nu = i$ components of Eq. (3.4) read

$$\begin{aligned} \delta\dot{U} = & -4\left[\delta\dot{S}_0 + 3H\delta\dot{S}_0 - S_0\ddot{\Phi} - 2\ddot{S}_0\Phi - 3\dot{S}_0(\dot{\Phi} + \dot{\Psi} + 2H\Phi) - 3H^2\delta S_0\right] \\ & + 12HS_0(\dot{\Phi} - 2\dot{\Psi} - 2H\Phi) - 2\frac{k^2}{a^2}(\delta S_0 + \delta\dot{S} - 4H\delta S - 2S_0\Phi), \end{aligned} \quad (3.19)$$

$$\delta U = -2\left[\delta\dot{S} + H\delta\dot{S} + 2\frac{k^2}{a^2}\delta S - 2(\dot{H} + 3H^2)\delta S + \delta\dot{S}_0 + 5H\delta S_0 - 2S_0(\dot{\Phi} + \dot{\Psi} + 4H\Phi) - 4\dot{S}_0\Phi\right]. \quad (3.20)$$

The evolution of the density contrast δ_m is known by solving Eqs. (3.10)-(3.11) and (3.14)-(3.20) for given k . In the $m \rightarrow 0$ limit, all the mass-dependent terms involving the perturbations δU , δS_0 , and δS in Eqs. (3.14)-(3.17) vanish to recover the General Relativistic behavior. When $m \neq 0$ the evolution of the gravitational potentials Φ and Ψ is subject to change, which affects the growth of δ_m through Eq. (3.13). Eliminating the terms $\dot{\Psi} + H\Phi$ from Eqs. (3.14) and (3.15), we obtain

$$\frac{k^2}{a^2}\Psi - \frac{m^2}{8}\delta\mathcal{F} = -4\pi G\rho_m\delta_m, \quad (3.21)$$

where

$$\delta\mathcal{F} \equiv \delta U + 4\delta\dot{S}_0 + 6H\delta S_0 + 6H\delta\dot{S} - 12H^2\delta S - 4S_0\dot{\Phi} - 4(2\dot{S}_0 + 3HS_0)\Phi. \quad (3.22)$$

In the $m \rightarrow 0$ limit, Eq. (3.21) reduces to the standard Poisson equation $(k^2/a^2)\Psi = -4\pi G\rho_m\delta_m$. In GR we have $\delta S = 0$ and $\Psi = \Phi$, so that the third term on the l.h.s. of Eq. (3.13) reads $(k^2/a^2)\Phi = -4\pi G\rho_m\delta_m$. This term works as a driving force for the growth of δ_m with the gravitational coupling characterized by G . In the presence of the mass term m , there is a modification to the gravitational constant G .

B. Evolution of perturbations on sub-horizon scales

Let us consider the perturbations relevant to the linear regime of galaxy clusterings. This corresponds to the wave numbers $0.01 h \text{ Mpc}^{-1} \lesssim k \lesssim 0.1 h \text{ Mpc}^{-1}$ [46], i.e.,

$$30 \lesssim k/H_0 \lesssim 300. \quad (3.23)$$

In the redshift range where the red-shift distortions of galaxies have been measured ($z \equiv 1/a - 1 \lesssim 2$), the modes (3.23) are deep inside the Hubble radius ($k/a \gg H$). Under a sub-horizon approximation we can ignore some of

the terms in the perturbation equations (3.14)-(3.20) (see e.g., Refs. [47–49]). We also note that the gravitational potentials Φ and Ψ are nearly constant during the deep matter era and they start to vary after the onset of the cosmic acceleration, so that $|\dot{\Phi}| \lesssim |H\Phi|$ and $|\dot{\Psi}| \lesssim |H\Psi|$ by today.

Under the sub-horizon approximation the dominant contributions to Eq. (3.18) should be the terms including k^2/a^2 , and hence

$$\delta U \simeq 8a_2(2\Psi - \Phi) = 8a_2(\Phi - 2m^2\delta S), \quad (3.24)$$

where in the second equality we used Eq. (3.16). For the validity of this approximation we also require that $(k^2/a^2)|\delta U| \gg |\Phi H\dot{U}|$, which can be interpreted as $k^2/(aH)^2 \gg |U|$ for $a_2 = 1/3$ and $|\dot{U}| \lesssim |HU|$. Since the today's value of $|U|$ is of the order of 10, the condition $k^2/(aH)^2 \gg |U|$ is satisfied for the wave numbers (3.23). We also note that Eq. (3.18) does not contain a large mass term exceeding k/a , so the oscillating mode induced by the second derivative $\delta\ddot{U}$ can be neglected relative to the mode (3.24).

We recall that the mass scale m is slightly smaller than H_0 , in which case k^2/a^2 is much larger than m^2 for the sub-horizon modes (3.23). From Eqs. (3.19) and (3.20) we can estimate the orders of the sub-horizon perturbations δS and δS_0 , as

$$|\delta S| \approx \frac{a^2}{k^2}|\Phi|, \quad |\delta S_0| \approx \frac{a^2 H}{k^2}|\Phi|. \quad (3.25)$$

From Eq. (3.16) it follows that

$$\left| \frac{\Psi}{\Phi} - 1 \right| = O(\epsilon_k), \quad \text{where} \quad \epsilon_k \equiv \frac{(ma)^2}{k^2}. \quad (3.26)$$

Since $\epsilon_k \ll 1$, the difference between Ψ and Φ is small. On using Eq. (3.25), the perturbation $\delta\mathcal{F}$ in Eq. (3.22) is approximately given by

$$\delta\mathcal{F} \simeq \delta U - 4S_0\dot{\Phi} - 4(2\dot{S}_0 + 3HS_0)\Phi. \quad (3.27)$$

The term $X = HS_0$ grows to the order close to $O(1)$ by today. Since $\delta U = O(\Phi)$ from Eq. (3.24), the perturbation $|\delta\mathcal{F}|$ is of the order of $|\Phi|$. From Eqs. (3.21) and (3.26) the third term on the l.h.s. of Eq. (3.13) can be estimated as

$$\frac{k^2}{a^2}\Phi \simeq -4\pi G_{\text{eff}}\rho_m\delta_m, \quad (3.28)$$

where the difference between the effective gravitational coupling G_{eff} and the gravitational constant G is

$$\left| \frac{G_{\text{eff}}}{G} - 1 \right| = O(\epsilon_k). \quad (3.29)$$

For the modes (3.23) the parameter ϵ_k is in the range $5 \times 10^{-6} \lesssim \epsilon_k \lesssim 5 \times 10^{-4}$, so G_{eff} is very close to G . The r.h.s. of Eq. (3.13) can be negligible relative to its l.h.s., and hence

$$\ddot{\delta}_m + 2H\dot{\delta}_m - 4\pi G_{\text{eff}}\rho_m\delta_m \simeq 0. \quad (3.30)$$

Numerically we have solved the full perturbation equations (3.10)-(3.11) and (3.14)-(3.20) for the initial conditions $\delta U(t_i) = \delta\dot{U}(t_i) = \delta S_0(t_i) = \delta\dot{S}_0(t_i) = \delta S(t_i) = \delta\dot{S}(t_i) = 0$ from the deep matter era. In spite of the presence of the second derivative $\delta\ddot{U}$ in Eq. (3.18), the term $(k^2/a^2)\delta U$ soon starts to balance with the term $8a_2(k^2/a^2)(2\Psi - \Phi)$ on the r.h.s. of Eq. (3.18). After that, the solutions can be well described by the analytic estimation given above. Numerically we also confirmed the accuracy of Eq. (3.29) and found that in practice $G_{\text{eff}}/G \simeq 1$ to better than 0.05% precision for the wave numbers in the range (3.23). This suggests that, apart from the difference of the background evolution, it is difficult to distinguish the NLMG model from the Λ CDM model for the perturbations relevant to large-scale structures. For the likelihood analysis presented in Sec. IV, we shall solve Eq. (3.30) by setting $G_{\text{eff}} = G$ together with the background equations of motion.

The observations of red-shift space distortions can place bounds on the quantity $f\sigma_8$, where $f \equiv \dot{\delta}_m/(H\delta_m)$ characterizes the growth rate of matter perturbations and σ_8 is the rms amplitude of δ_m at the comoving $8 h^{-1}$ Mpc scale [46]. In Fig. 3 we plot $f\sigma_8$ versus the redshift z for the NLMG model with $a_1 = 0$ and $a_2 = 1/3$ as well as for the Λ CDM model. The today's values of Ω_m and σ_8 are chosen to be $\Omega_m^{(0)} = 0.3$ and $\sigma_{8,0} = 0.8$, respectively. The

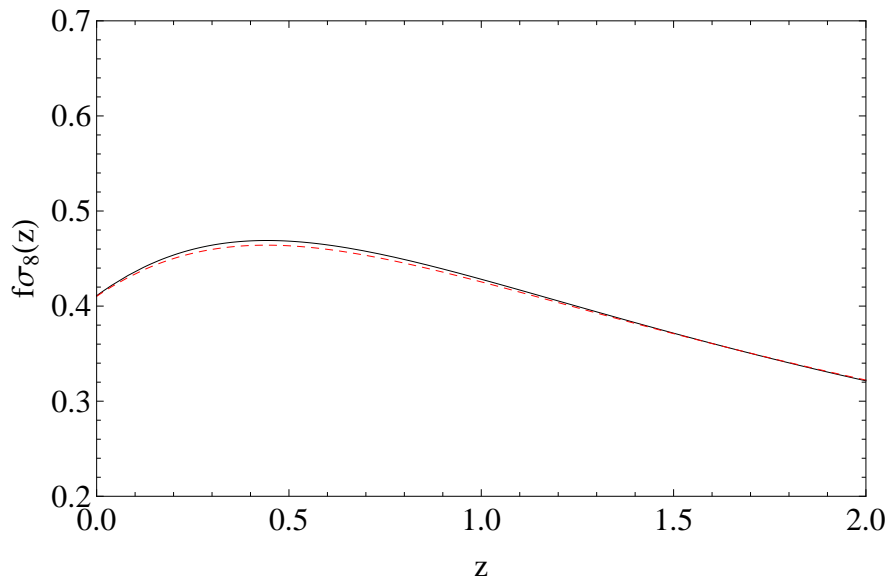


FIG. 3: The evolution of $f\sigma_8(z)$ for the NLMG model with $a_1 = 0$ and $a_2 = 1/3$, compared to the Λ CDM model in the redshift range $z \in [0, 2]$. The two lines correspond to $k = 30H_0$ (solid black line) and the Λ CDM model (dashed red line) for $\Omega_m^{(0)} = 0.3$ and $\sigma_{8,0} = 0.8$. We plot $f\sigma_8$ in this redshift range as this is where the current data exist.

solid black curve in Fig. 3 corresponds to the wave number $k = 30H_0$, but we confirmed that the evolution of $f\sigma_8$ for $k > 30H_0$ is similar to that for $k = 30H_0$.

Since G_{eff} is very close to G for the modes (3.23), the growth rate of matter perturbations in the NLMG model is similar to that in the Λ CDM model. The main reason of the small difference seen in Fig. 3 is that the background evolution of w_{DE} is different. This is similar to what happens for the constant w_{DE} models in the framework of GR [50]. For a practical purpose, the evolution of $f\sigma_8$ in the NLMG model can be known in good accuracy by solving Eq. (3.30) with $G_{\text{eff}} = G$.

IV. OBSERVATIONAL CONSTRAINTS

In this section we will confront the NLMG model (2.1) characterized by $a_1 = 0$ and $a_2 = 1/3$ with the latest cosmological data and study whether they can be distinguished from the Λ CDM model.

A. The data

In order to constrain the NLMG model, we use the same numerical code² and the same data of SnIa, BAO and growth-rate as those in Refs. [51, 52], so we refer the readers to the aforementioned references for detail. We also employ the correlation matrix of the Planck CMB shift parameters (l_a, \mathcal{R}, z_*) presented in Ref. [53]. These three parameters are related to the background quantities such as $\Omega_m^{(0)}$, $\Omega_b^{(0)}$, and h . They can efficiently summarize the CMB information on dark energy in a model-independent way [54].

Compared to Refs. [51, 52], there is a difference in the analysis of the growth-rate data. Instead of using the well known $\gamma(z)$ parameterization and modeling the growth-rate as $f(z) = \Omega_m(z)^{\gamma(z)}$, we directly fit the numerical solution of the perturbation equations. Regarding the data of the growth rate given in Table I of Ref. [51], they are based on the *WiggleZ*, SDSS, 2dF, PSCz, VVDS, 6dF, 2MASS and BOSS galaxy surveys. The data themselves are given in terms of $f(z)\sigma_8(z)$. It should be stressed that the main benefit of using $f(z)\sigma_8(z)$, instead of just $f(z)$, is that the former is directly related to the power spectrum of peculiar velocities of galaxies.

² General minimization and MCMC cosmological codes can be found freely available at: <http://www.uam.es/savvas.nesseris>

Model	$\Omega_m^{(0)}$	$\Omega_b^{(0)} h^2$	$\sigma_{8,0}$	χ_{bf}^2	AIC	$ \Delta\text{AIC} $
$h = 0.673$						
ΛCDM	0.328 ± 0.002	0.0234 ± 0.0002	0.735 ± 0.019	583.470	589.470	0
NLMG	0.334 ± 0.002	0.0223 ± 0.0002	0.726 ± 0.019	585.570	591.570	2.100
$h = 0.738$						
ΛCDM	0.252 ± 0.002	0.0249 ± 0.0002	0.789 ± 0.021	599.620	605.620	10.011
NLMG	0.257 ± 0.002	0.0245 ± 0.0002	0.775 ± 0.020	589.609	595.609	0

TABLE I: Statistical results of the overall likelihood analysis: The first column indicates the model, while the second, third, and fourth columns provide the $\Omega_m^{(0)}$, $\Omega_b^{(0)} h^2$, and $\sigma_{8,0}$ best-fit values. The last three columns present the goodness-of-fit statistics (χ_{bf}^2 , AIC and $\Delta\text{AIC}_{i,j} = \text{AIC}_i - \text{AIC}_j$). All the error estimates come from the inverse of the Fisher matrix. The upper part of Table shows the values for the Planck prior $h = 0.673$ [2], while the lower half the corresponding values for the Riess *et al.* prior $h = 0.738$ [55].

B. Fitting method and model comparisons

As mentioned in the previous section, the mass m is known from Eq. (2.26) and it is not a free parameter of the theory. We determine m such that the system of the background equations of motion is consistent, i.e., the value initially used for the solution has to be the same as the one derived from Eq. (2.26). Since the results depend on $\Omega_m^{(0)}$, we implement an iterative algorithm in which the value of m is found for each value of $\Omega_m^{(0)}$ via Eq. (2.26) to check the consistency of Eqs. (2.18)-(2.23). These values are also saved and used later on to simplify and speed up the fitting procedure. Therefore, the final set of parameters employed in the minimization is $(\Omega_m^{(0)}, \Omega_b^{(0)} h^2, \sigma_{8,0})$, where $\Omega_b^{(0)}$ is the today's baryon density parameter. This situation is analogous to the ΛCDM model.

We compute the total chi square

$$\chi^2 = \chi_{\text{SnIa}}^2 + \chi_{\text{BAO}}^2 + \chi_{\text{CMB}}^2 + \chi_{\text{growth}}^2, \quad (4.1)$$

where each term on the r.h.s. is derived by fitting with the SnIa, BAO, CMB, and growth-rate data, respectively, along the line of Refs. [51, 52]. The best-fit corresponds to model parameters for which χ^2 takes a minimum value χ_{bf}^2 . We will also consider the same parameters $(\Omega_m^{(0)}, \Omega_b^{(0)} h^2, \sigma_{8,0})$ in the ΛCDM model and evaluate the total chi square for the comparison with the NLMG model.

Following Ref. [53], we discuss the effect of the H_0 prior on the results. The Planck team essentially pinned down the parameter $\Omega_m^{(0)} h^2$ to very high precision, so changing h also affects $\Omega_m^{(0)}$, since $\delta(\Omega_m^{(0)} h^2) \simeq 0$ or equivalently $\delta \ln \Omega_m^{(0)} \simeq -2\delta \ln h$. The latter implies that, while fitting the data, increasing h forces $\Omega_m^{(0)}$ to lower values and vice versa. However, due to the degeneracies in the CMB+BAO data, a lower value of $\Omega_m^{(0)}$ implies a more negative dark energy equation of state, i.e. $w_{\text{DE}} < -1$. In simple terms, increasing h reduces $\Omega_m^{(0)}$ and forces w_{DE} to more negative values and vice versa. Therefore, the value h that we choose is important in the rest of the analysis especially since, as mentioned before, the NLMG model has a corresponding equation of state w_{DE} between -1.1 and -1.04 . In order to accommodate the cases with different values of H_0 , we will test some priors on h : (i) the Planck best-fit: $h = 0.673$ [2], and (ii) the best-fit $h = 0.738$ derived by the direct measurement of H_0 [55], and (iii) other four values of h ranging $0.673 < h < 0.738$.

We will also consider the Akaike information criterion (AIC) [56] as in Refs. [51, 52]. The AIC is defined, for the case of Gaussian errors, as:

$$\text{AIC} = \chi_{\text{bf}}^2 + 2\ell, \quad (4.2)$$

where ℓ is the number of free parameters. A smaller value of the AIC indicates a better fit to the data. In order to effectively compare two different models, we need to estimate the differences $\Delta\text{AIC}_{1,2} = \text{AIC}_1 - \text{AIC}_2$ for the two models 1 and 2. Since $\ell = 3$ in both the NLMG and the ΛCDM models, the difference of AIC between the two models is actually equivalent to that of χ_{bf}^2 between them. The larger the value of $|\Delta\text{AIC}|$, the higher the evidence against the model with a larger value of AIC, with a difference $|\Delta\text{AIC}| \gtrsim 2$ indicating weak evidence and $|\Delta\text{AIC}| \gtrsim 6$ indicating a stronger evidence in favor of the model with smaller AIC, while a value $\lesssim 2$ indicates consistency among the two comparison models. The AIC penalizes models with more parameters, but it should be stressed that these numbers are provided only as a rule of thumb and they should be used with caution [57].

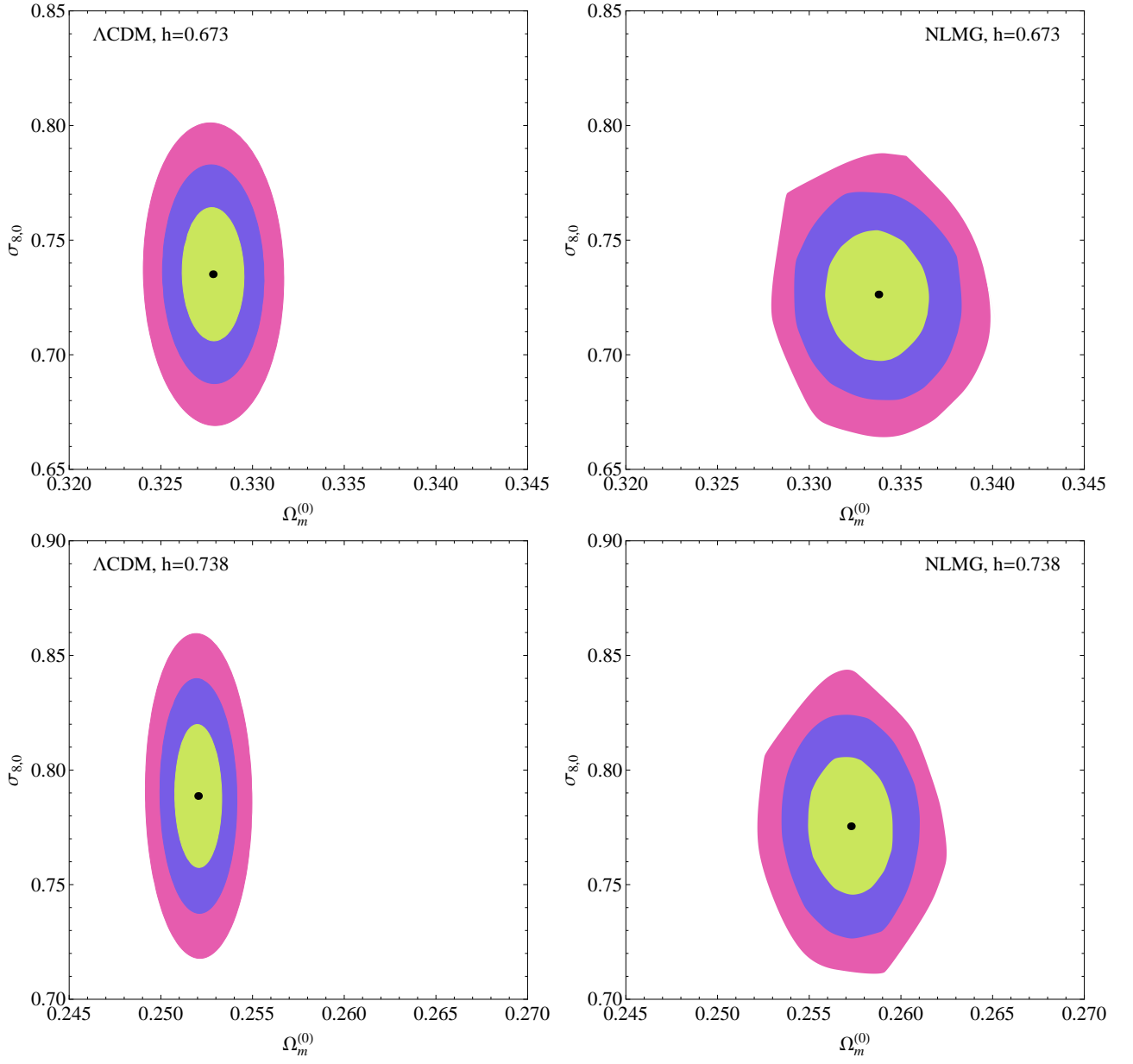


FIG. 4: The 1, 2 and 3σ contour plots for the Λ CDM (left) and the NLMG (right) for $a_1 = 0$ and $a_2 = 1/3$ in the $(\Omega_m^{(0)}, \sigma_{8,0})$ parameter space. The top row shows the contours for $h = 0.673$ and the bottom row for $h = 0.738$.

C. Results

In Table I we show the best-fit values of $\Omega_m^{(0)}$, $\Omega_b^{(0)}h^2$, and $\sigma_{8,0}$ both for the Λ CDM and the NLMG models with $a_1 = 0$ and $a_2 = 1/3$ with two different values of h . In Figs. 4 and 5 we also plot the 1, 2 and 3σ observational contours in the $(\Omega_m^{(0)}, \sigma_{8,0})$ and $(\Omega_m^{(0)}, \Omega_b^{(0)}h^2)$ planes, respectively, for the two models with $h = 0.673$ and $h = 0.738$. For the Planck prior $h = 0.673$ the minimum chi square in the NLMG is found to be $\chi_{\text{bf}}^2 = 585.570$, which is slightly larger than that in the Λ CDM ($\chi_{\text{bf}}^2 = 583.470$). Since the difference of AIC between the two models is $|\Delta\text{AIC}| = 2.100$, either of them is not particularly favored over the other.

For $h = 0.673$, if we divide the minimum chi square as Eq. (4.1), each contribution is given by $\chi_{\text{bf},\text{SnIa}}^2 = 566.478$, $\chi_{\text{bf},\text{BAO}}^2 = 7.84942$, $\chi_{\text{bf},\text{CMB}}^2 = 3.67004$, $\chi_{\text{bf},\text{growth}}^2 = 7.57215$ in the NLMG and $\chi_{\text{bf},\text{SnIa}}^2 = 568.399$, $\chi_{\text{bf},\text{BAO}}^2 = 6.62794$, $\chi_{\text{bf},\text{CMB}}^2 = 0.829078$, $\chi_{\text{bf},\text{growth}}^2 = 7.61382$ in the Λ CDM. The growth data do not provide any significant difference between the two models as expected, whereas the SnIa data alone prefer the NLMG to the Λ CDM. The $\chi_{\text{bf},\text{CMB}}^2$ in

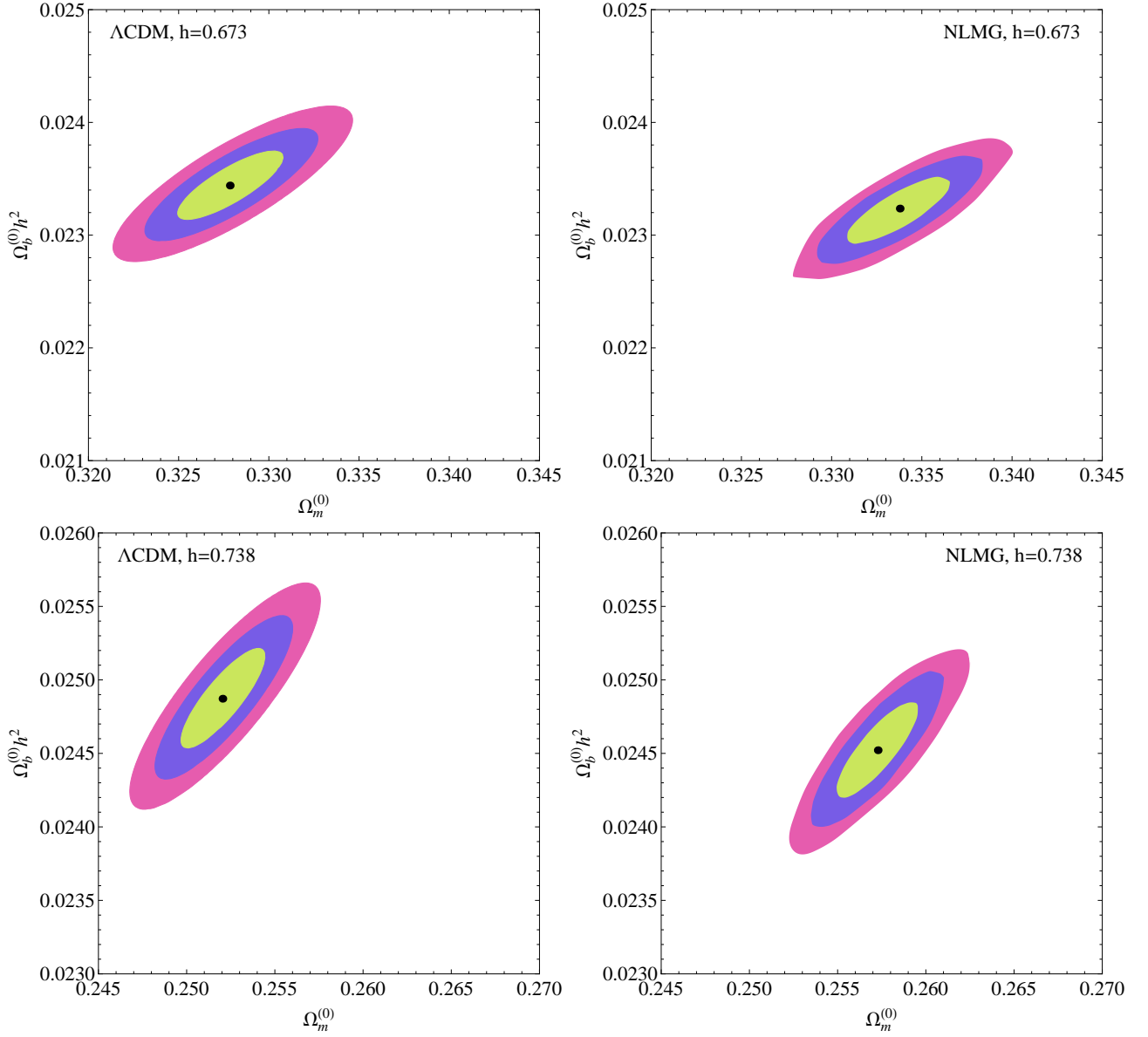


FIG. 5: The 1, 2 and 3 σ contour plots for the Λ CDM (left) and the NLMG (right) for $a_1 = 0$ and $a_2 = 1/3$ in the $(\Omega_m^{(0)}, \Omega_b^{(0)} h^2)$ parameter space. The top row shows the contours for $h = 0.673$ and the bottom row for $h = 0.738$.

the NLMG is larger than that in the Λ CDM with the large difference 2.842, which is the main reason why the total χ_{bf}^2 in the former exceeds that in the latter for $h = 0.673$.

For $h = 0.738$ the best-fit NLMG and Λ CDM models correspond to $\chi_{\text{bf}}^2 = 589.609$ and $\chi_{\text{bf}}^2 = 599.620$, respectively. Hence the NLMG is significantly favored over the Λ CDM with the difference $|\Delta\text{AIC}| = 10.011$. In this case, the contributions to the minimum chi square are $\chi_{\text{bf,SnIa}}^2 = 565.929$, $\chi_{\text{bf,BAO}}^2 = 5.38663$, $\chi_{\text{bf,CMB}}^2 = 10.3146$, $\chi_{\text{bf,growth}}^2 = 7.97966$ in the NLMG and $\chi_{\text{bf,SnIa}}^2 = 564.010$, $\chi_{\text{bf,BAO}}^2 = 6.96224$, $\chi_{\text{bf,CMB}}^2 = 20.5558$, $\chi_{\text{bf,growth}}^2 = 8.08938$ in the Λ CDM. Therefore, both the CMB and BAO data prefer the NLMG to the Λ CDM. For larger h , the constrained regions in Figs. 4 and 5 shift toward smaller values of $\Omega_m^{(0)}$. For $\Omega_m^{(0)}$ around $0.25 \sim 0.26$, the models with $w_{\text{DE}} < -1$ are favored over the Λ CDM. We note that the growth data do not provide any significant difference between the two models. The change of the bounds on $\sigma_{8,0}$ relative to the case $h = 0.673$ (seen in Fig. 4) mainly comes from the shift of $\Omega_m^{(0)}$.

We also perform a more extensive analysis with intermediate values of the H_0 prior as well. In Fig. 6 we show on the left panel the values of the best-fit χ^2 as a function of the prior H_0 for both the NLMG (solid line) and the Λ CDM (dashed line) models, respectively. On the right panel we plot the difference of χ_{bf}^2 between the NLMG and

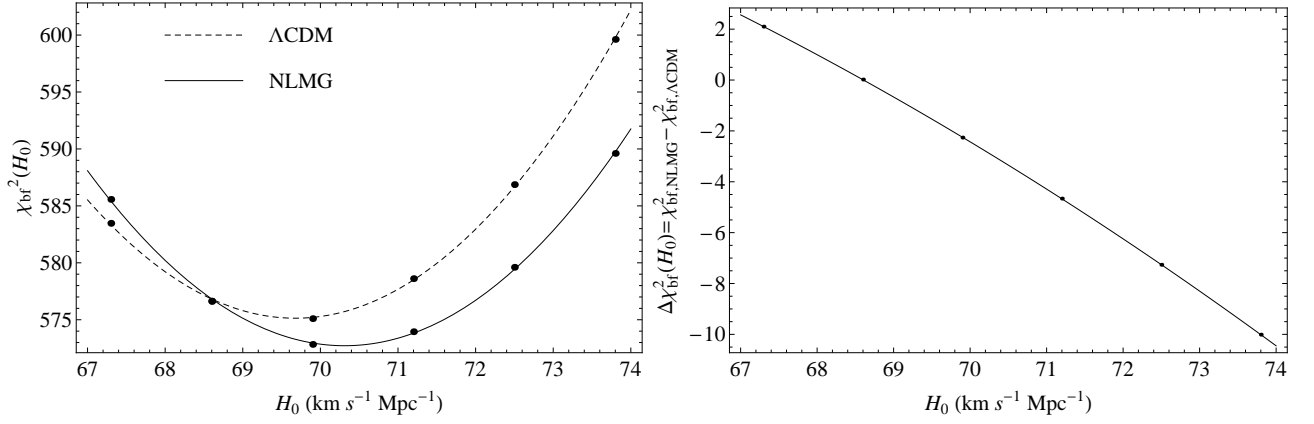


FIG. 6: Left: The value of the best-fit χ^2 as a function of the prior H_0 for both the NLMG (solid line) and the ΛCDM (dashed line) models, respectively. Right: The difference in the best-fit models between the NLMG and the ΛCDM , fitted by a quadratic function (see the text). Clearly, higher values of the H_0 prior strongly prefer the NLMG compared to the ΛCDM .

the ΛCDM models. As a function of h , χ^2_{bf} has a minimum around $h_1 = 0.703$ in the NLMG and $h_2 = 0.697$ in the ΛCDM . We adopt the following quadratic functions expanded around h_1 and h_2 respectively:

$$\begin{aligned}\chi^2_{\text{bf,NLMG}}(h) &= \chi^2_{\text{bf,NLMG}}(h_1) + \frac{1}{2}\partial_h^2\chi^2_{\text{bf,NLMG}}(h_1)(h - h_1)^2 + \dots, \\ \chi^2_{\text{bf,}\Lambda\text{CDM}}(h) &= \chi^2_{\text{bf,}\Lambda\text{CDM}}(h_2) + \frac{1}{2}\partial_h^2\chi^2_{\text{bf,}\Lambda\text{CDM}}(h_2)(h - h_2)^2 + \dots,\end{aligned}\quad (4.3)$$

where we used the fact that at the minimum the first derivative is zero. Taking the difference, we obtain

$$\Delta\chi^2_{\text{bf}}(h) \equiv \chi^2_{\text{bf,NLMG}}(h) - \chi^2_{\text{bf,}\Lambda\text{CDM}}(h) = b_1 + b_2h + b_3h^2 + \dots, \quad (4.4)$$

where the constants (b_1, b_2, b_3) are related to the various terms of Eqs. (4.3). As we see in Fig. 6, this fit shows good agreement with χ^2_{bf} derived for some discrete values of h . For $h > 0.686$, χ^2_{bf} in the NLMG is smaller than that in the ΛCDM . In particular, for $h > 0.70$, the NLMG is favored over the ΛCDM according to the AIC.

When we consider the general case with a small but non-vanishing a_1 , the instability of the field V leads to the dark energy equation of state much smaller than -1 . We have carried out the joint data analysis based on the SNIa, CMB, and BAO data for the NLMG model with $a_1 = 0.01$ and $a_2 = 1/3$ and for the ΛCDM model. For $h = 0.738$ we find that the best-fits correspond to $\chi^2_{\text{bf}} = 727.802$ in the NLMG and $\chi^2_{\text{bf}} = 591.528$ in the ΛCDM , respectively. The difference of AIC from the model $a_1 = 0$ and $a_2 = 1/3$ is $|\Delta\text{AIC}| \sim 140$, so the models with $a_1 \neq 0$ are significantly disfavored from the data.

V. CONCLUSIONS

In this paper we studied cosmological perturbations and observational constraints on the NLMG model. Our analysis of the background cosmology covers the two models given by Eqs. (1.1) and (1.2). We dealt with (2.1) as an effective classical equation of motion for discussing the cosmology relevant to dark energy. The issues of ghosts and ultraviolet completion should be addressed in a more fundamental theory with a Lagrangian implementing quantum and classical averaging.

We derived the background equations of motion from (2.1) on the flat FLRW background. For the models with $a_1 \neq 0$ there is an instability for the field V induced by the $-8V$ term on the l.h.s. of Eq. (2.19). In this case the mass m is required to be much smaller than H_0 to avoid the early onset of the cosmic acceleration. Since the dark energy equation of state significantly deviates from -1 , the models with $a_1 \neq 0$ are strongly disfavored from the joint analysis of the SNIa, CMB, and BAO data.

For the models with $a_1 = 0$ the r.h.s. of Eq. (2.19) vanishes, so that the field V does not grow for the appropriate initial conditions (2.27). In order for the dark energy density ρ_{DE} to be positive, we found that the parameter a_2 has to satisfy the condition $a_2 > 0$. The dark energy equation of state w_{DE} in the deep matter era can be estimated as Eq. (2.32), which shows good agreement with the numerically integrated solution ($-1.1 < w_{\text{DE}} < -1.04$ for $-4 < \ln a < 0$).

Expanding the field equations of motion and the metric to first order in perturbations about the flat FLRW background, we derived the full equations of cosmological perturbations for the NLMG model (1.2). The behavior of perturbations is also estimated for the modes relevant to galaxy clusterings. We found that the effective gravitational coupling G_{eff} is very close to the gravitational constant G for sub-horizon perturbations characterized by the wave numbers (3.23). Therefore, the evolution of $f\sigma_8$ is similar to that in the Λ CDM model (see Fig. 3). In this sense, the current growth-rate measurement alone is not able to distinguish between the NLMG and the Λ CDM models.

We compared the NLMG model (1.2) against the latest cosmological observations, including SNIa, BAO, CMB, and red-shift space distortions. Since the mass m is not a free parameter, we developed an iterative algorithm to compute m for each value of $\Omega_m^{(0)}$ via Eq. (2.26) and checked the consistency of Eqs. (2.18)-(2.23). The mass m in the NLMG plays a similar role to the cosmological constant Λ in the Λ CDM, such that the number of free parameters in the two models is the same. For the sake of comparison we considered the same parameters ($\Omega_m^{(0)}, \Omega_b^{(0)}h^2, \sigma_{8,0}$) in both models.

The observational constraints on the NLMG model mainly come from the background expansion history rather than the growth history. The dark energy equation of state varies slowly from the deep matter era to today around $-1.1 \lesssim w_{\text{DE}} \lesssim -1.04$. Since w_{DE} is approximately constant in the past, the situation is quite similar to the case of the constant w_{DE} models studied in Ref. [53]. The likelihood results depend on the value of the H_0 prior due to the degeneracies of the CMB parameters.

We computed the chi squares in both the NLMG and the Λ CDM models for several different values of H_0 ranging from the Planck best-fit $h = 0.673$ [2] to the Riess *et al.* best-fit $h = 0.738$ [55]. The results of our analysis are presented in Table I and Figs. 4-6. For $0.67 \lesssim h \lesssim 0.70$ the AIC shows that the NLMG and the Λ CDM models are statistically comparable, but for $h \gtrsim 0.70$ the NLMG model is strongly favored over the Λ CDM model. We hope that future observations will pin down the values of h to exquisite accuracy, clarifying whether the NLMG model is really preferred to the models with $w_{\text{DE}} \geq -1$.

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